Towards a Fully 3D Transient-Dedicated Error Criterion

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***Abstract* — In order to improve the finite element modeling of macroscopic eddy currents, a quadratic energy-based error criterion is obtained from a thermodynamic description of electromagnetism. Attention is first paid on the analytical derivation of the criterion in a general sense — *i.e.* without any assumption about the potential formulation and including possible body motion — especially to validate its relevance in 2D or 3D and to stress its independence from the formulation or the kind of application. A 3D-validation is given on an induction heating process: the conservation of the electromagnetic power is assessed locally and, as expected, the ill-checked elements in the skin depth are highlighted.**

I. INTRODUCTION

Eddy currents are at the origin of losses and signal distortions in power electrical devices. In order to address their considerable impacts on both the energy efficiency and the performance requirement, eddy currents modeling and its accuracy are discussed from a thermodynamic approach.

While some former attempts, *e.g.* [1][2], are focused on the consistency of the magnetostatic resolution, the proposed error criterion is fully dedicated to the evaluation of the dynamic aspect in massive conductor where the skin effect occurs. It is independent of the formulation, and allows taking body motion into account.

A presentation of the variational approach of electromagnetism is first proposed. Then the error criterion is naturally derived. Finally, some numerical results obtained by Finite Element computation on an induction heating process are presented.

II. VARIATIONAL FORMULATION

Classically, thermodynamic approaches of electromagnetism do not consider any extension towards time-varying regimes [3][4][5]. Whereas some improvements are summarized in [6] for steady states regimes, no general contribution is available for transient. Denoting as a general rule in this work, variational parameters or functional thanks to *italic* fonts whereas roman ones specify their values at the minimum, the magnetodynamic behavior of any electrical system is derived from the functional [7]:

$$-\mathrm{P}_{\mathrm{mech}}-\frac{\mathrm{dG}}{\mathrm{d}t}=\min_{\boldsymbol{H},\boldsymbol{E}}\left(\int_C \sigma^{-1}(\mathbf{curl}\boldsymbol{H})^2\,\mathrm{d}^3r+\frac{\mathrm{d}}{\mathrm{d}t}\int(\mathbf{B}\cdot\boldsymbol{H}+\mathbf{D}\cdot\boldsymbol{E})\mathrm{d}^3r\right) \quad (1)$$

where the functional in the RHS exhibits:

- the magnetic field $\boldsymbol{H}$ related to free and displacement currents according to the Maxwell-Ampere equation. The quasi-static approximation enforces $\mathbf{D}\equiv 0$ in conductors;
- the Joule losses P_{J} monitored in conductors. This term is even to respect invariance of losses with the inversion of time (σ^{-1} is the resistivity);
- the variation with time of the electromagnetic energy coupling the field with the generator I and the mass V_0;
- the magnetic $\mathbf{B}(\boldsymbol{h})$ and electrostatic $\mathbf{D}(\boldsymbol{e})$ behavior laws derived from thermostatic equilibrium of the Gibbs potential:

$$G(\mathrm{I},\mathrm{V}_0)=\int\left(\int_0^{\boldsymbol{H}}(-\boldsymbol{B})\cdot\delta\boldsymbol{h}+\int_0^{\boldsymbol{E}}(-\boldsymbol{D})\cdot\delta\boldsymbol{e}\right)\mathrm{d}^3r \quad (2)$$

Extending the electric field in the conductor according to Ohm's law $\mathbf{E}=\sigma^{-1}\mathbf{J}-\mathbf{V}\times\mathbf{B}$, Faraday's law $\mathbf{curl}\,\mathbf{E}=-\partial_t\mathbf{B}$ may be viewed as a local result of the global trend towards reversibility expressed by (1) [8]. This striking property provides a thermodynamic oriented insight of the variational theory of electromagnetism [9]. Hence, the functional (1) balances the variations with time of the co-energy (–G) and the mechanical power received from the field by the actuators ($-\mathrm{P}_{\mathrm{mech}}$).

In order to consider sub-systems for design purpose, it is convenient to introduce the electrical power:

$$P_{\mathrm{elec}}(\Omega)=-\oint_{\partial\Omega}(\boldsymbol{E}\times\boldsymbol{H})\cdot\mathbf{n}\,\mathrm{d}^2r \quad (3)$$

After some calculations, it follows:

$$\begin{aligned}P_{\mathrm{elec}}(\Omega)&=-\int_\Omega(\mathbf{curl}\boldsymbol{E}+\partial_t\boldsymbol{B})\cdot\boldsymbol{H}\,\mathrm{d}^3r\\&+\int_\Omega(\mathbf{curl}\boldsymbol{H}-\boldsymbol{J}-\partial_t\boldsymbol{D})\cdot\boldsymbol{E}\,\mathrm{d}^3r\\&+\int_{C\subset\Omega}\boldsymbol{J}\cdot\left(\boldsymbol{E}-\sigma^{-1}\boldsymbol{J}+\mathbf{V}\times\boldsymbol{B}\right)\mathrm{d}^3r\\&+P_{\mathrm{J}}(\Omega)+\frac{\mathrm{d}F}{\mathrm{d}t}(\Omega)\\&-\sum_i\oint_{\partial\Omega_i}\left([\boldsymbol{E}\times\boldsymbol{H}]\cdot\mathbf{n}-\left[\int_0^{\boldsymbol{B}}\boldsymbol{H}\cdot\delta\mathbf{b}+\int_0^{\boldsymbol{D}}\boldsymbol{E}\cdot\delta\mathbf{d}\right](\mathbf{V}_i\cdot\mathbf{n})\right)\mathrm{d}^2r\\&+\int_{C\subset\Omega}\mathbf{V}\cdot(\boldsymbol{J}\times\boldsymbol{B})\mathrm{d}^3r\end{aligned} \quad (4)$$

where F is the Helmholtz's potential and [·] denotes the discontinuity occurring at the interfaces $\partial\Omega_i\subset\Omega$. At the minimum of the functional (1), the Maxwell equation set and Ohm's law are checked so that:

- the first three residual terms vanish in (4). After some tedious calculations on the motion induced-conductor interface discontinuities, the two last terms provide the mechanical power supplied by the field:

$$\begin{aligned}\mathrm{P}_{\mathrm{mech}}(\Omega)&=\sum_i\oint_{\partial\Omega_i}(\mathbf{n}\cdot\mathbf{V})[\mathbf{D}\times\mathbf{B}]\cdot\mathbf{V}_i\,\mathrm{d}^2r+\sum_i\int_{C_i}(\mathbf{J}\times\mathbf{B})\cdot\mathbf{V}_i\,\mathrm{d}^3r\\&+\sum_i\oint_{\partial\Omega_i}\left([(\mathbf{B}\cdot\mathbf{n})\mathbf{H}+(\mathbf{D}\cdot\mathbf{n})\mathbf{E}]-\left[\int_0^{\mathbf{H}}\mathbf{B}\cdot\delta\mathbf{h}+\int_0^{\mathbf{E}}\mathbf{D}\cdot\delta\mathbf{e}\right]\cdot\mathbf{n}\right)\cdot\mathbf{V}_i\,\mathrm{d}^2r\end{aligned} \quad (5)$$

  where the first term denotes a vanishing "impulsion" term within the quasi-static approximation; the second one is related to the power of the Laplace's force; and the third one gathers the switching reluctance effects occurring at the various interfaces of the domain Ω. For conductors with linear magnetic behavior law, these three contributions may be lumped in the Maxwell's stress tensor. As a result, the relation (4) matches the integral form of the Poynting's conservation equation. Hence,

- the contribution of Ω to (1) reads:

$$-\mathrm{P}_{\mathrm{mech}}(\Omega)+\mathrm{P}_{\mathrm{elec}}(\Omega)-\frac{\mathrm{d}G}{\mathrm{d}t}(\Omega) \tag{6}$$

The previous approach addresses a thermodynamic-oriented justification of the Finite Element Method, which consists in building an approximation of the stationary conditions expressed at Eqs. (1) and (2) but with a finite number of degrees of freedom. By inspection of (4), the consistency of any transient solution with energy conservation may be assessed through the local deviation of the Poynting's equation:

$$\varepsilon(\Omega)=P_{\mathrm{elec}}(\Omega)-P_{\mathrm{J}}(\Omega)-\frac{\mathrm{d}F}{\mathrm{d}t}(\Omega)-\mathrm{P}_{\mathrm{mech}}(\Omega) \tag{7}$$

Indeed, strictly enforcing two relations among Maxwell-Ampere or Maxwell-Faraday equations and Ohm's law, the error criterion (7) highlights the elements where the remaining one is ill-checked.

III. NUMERICAL RESULTS

Due to its quadratic definition, it should be noticed that the criterion (7) is dedicated to all kinds of formulation even though, restoring locally the state variables of the whole system, current-based formulations are naturally compliant with the thermodynamic approach. Nevertheless, the error criterion (7) was first implemented in 2D-transient – *i.e.* within the magnetic vector potential formulation – to check the convergence of the accuracy of the eddy-current computation in a Thomson effect device [10]. In spite of significant eddy currents therein, the latter could not provide a general situation because magnetic parts were missing.

In order to check the behavior of the criterion in 3D, an induction heating process was modeled within a time-harmonic regime (Fig.1). As the mesh is refined in the skin depth, the error criterion is decreased therein. Global convergence of functionals (2) and (6) will be provided in the extended paper. To do so, electric field calculation has to be post-processed in the air region as the result of a magnetostatic-like problem (where $-\partial_t\mathbf{B}$ acts as a source term therein and $[\mathbf{E}]\times\mathbf{n} = 0$ is enforced at the conductor boundaries ∂C).

IV. CONCLUSION

The energy-based error criterion allows refining the conducting regions where the mesh is too coarse, with respect to the skin effect occurring therein. In addition to the error criterion, the thermodynamic approach provides the functionals from which the global convergence should be evaluated. Subsequent post-processing development should be considered to extend the criterion in the air region and fully address, by an iterative coupling with a meshing procedure, an adaptative meshing technique.

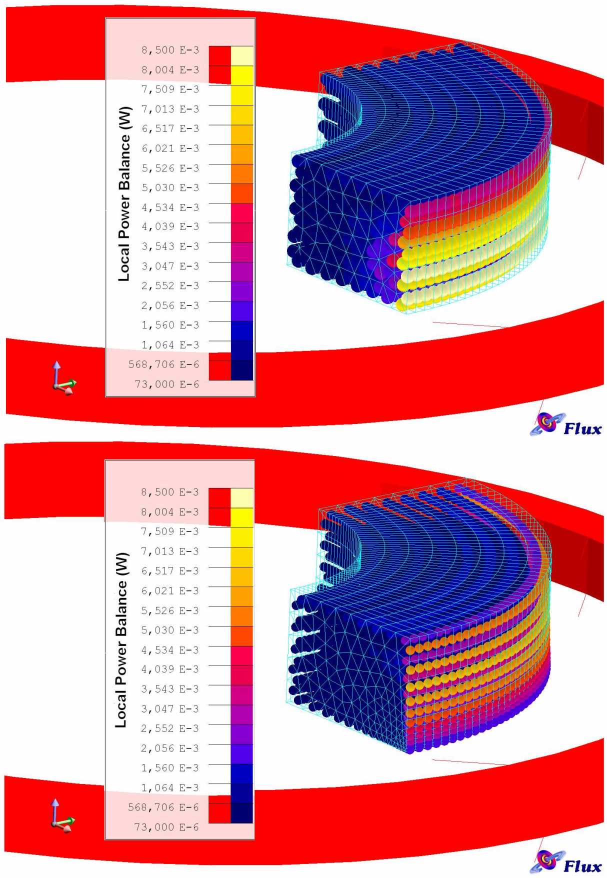

Fig. 1. Induction heating process: Error criterion (7) in the conducting region for the initial (top) and the refined (bottom) meshes. As the mesh is refined in the skin depth, notice the spread reduction of the criterion (7) therein.

V. REFERENCES

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